國立政治大學　應用數學系 100 學年度　第一學期　學科　考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM　　Page 1/Total 1

考試科目 Course	組合學	開課系級 Dept. & Class	研究所	日　期 Date, Period	100 年 09 月 19 日 上午 09:00~12:00	試題編號 Course No.	

本試卷共有 6 個題目。
碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。
博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

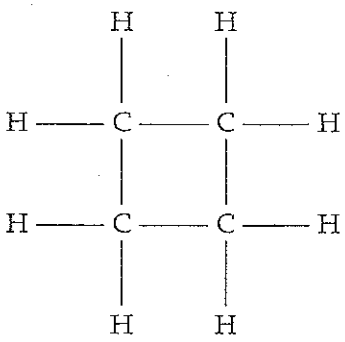
★ Show *all* your work for credits!

1. Suppose α is an irrational real number. Then there are infinitely many rational numbers p/q such that
$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{q^2}.$$

2. We say that a rooted tree is *strictly binary* if every parent vertex has exactly two children. How many strictly binary trees are there with k parent vertices? (Do not take symmetry into account: If two trees are mirror images of one another, count both configurations.)

3. If $2 \le p' \le p$ and $2 \le q' \le q$, then prove that $R(p',q') \le R(p,q)$ where $R(\cdot,\cdot)$ is the Ramsey number associated with integers. Also, prove that equality holds if and only if $p' = p$ and $q' = q$.

4. A cyclobutane is a hydrocarbon constructed of 4 carbon atoms arranged cyclically with 2 hydrogen atoms attached to each carbon, as illustrated in the following:



   (a) How many isomers can be obtained by replacing 2 hydrogens with nitrogen and 3 with oxygen?
   (b) Find the number of isomers with 3 hydrogens.

5. Let $n > 1$ be an integer. A *conference matrix* M of order n is an $n \times n$ matrix with 0's on the diagonal and $+1$ or -1 in all other positions, and with the property
$$MM^t = (n-1)I_n$$
where I_n is the identity matrix of order n and M^t is the transpose of M.
   (a) Show that n must be even.
   (b) Show that permuting rows and columns and multiplying rows and columns by -1, we can obtain a matrix that is symmetric if $n \equiv 2 \bmod 4$ and antisymmetric if $n \equiv 0 \bmod 4$.

6. Let $GF(4) = \{0, 1, \omega, \overline{\omega}\}$, where $\omega^3 = 1$ and $\overline{\omega} = \omega^2$.
   (a) Give a parity-check matrix for the Hamming single error-correcting code C of length 5 over $GF(4)$.
   (b) Give a generator matrix for C.
   (c) What is the minimum weight of C? Give reasons.

本考試：■ 不需使用簡易計算機，□ 使用簡易計算機　　←請出題老師勾選，謝謝！

In the Problem 4, you must graphically visualize the cyclobutane in the 3 dimensional space.