國立政治大學 應用數學系 103學年度第一學期　學科 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM　　Page (/Total

考試科目 Course	組合學	開課系級 Dept, & Class	研究所	日期 Date, Period	103年9月22日 上午9:00~12:00	試題編號 Course No.	

本試卷共有6個題目，

碩士班：請選5題作答，每題20分，請在答案卷最前面註明所選的5題，否則依學生作答之前5題計分。

博士班：6題全做答，每題17分，超過100分則以100分計。 Show all your work.

1. Let a_n be the number of nonnegative integral solutions of the equation $z_1+2z_2+z_3+z_4=n$ with the restrictions that $z_1\geq 3$ and $z_3\leq 3$. Find (1) a generating function for a_n (2) a formula for a_n (3) a_5.

2. Find a formula for the number of sequences of n digits $(0,1,2,\ldots,9)$ with even number of zeros.

3. How many ways can the faces of a cube be colored with three faces red, two white, and one blue?

4. If the graph G has $2n$ vertices and no triangles, then show that G cannot have more than n^2 edges.

5. A graph is color critical if the removal of any vertex decreases the graph's chromatic number. Show that every k-chromatic color critical graph G has the following properties: (a) G is connected. (b) Every vertex of G has degree $\geq k-1$. (c) G has no vertex whose removal disconnects G.

6. Determine the number of permutations of $\{1,2,\ldots,8\}$ in which no even integer is in its natural position.

本考試：☑ 不需使用簡易計算機，☐ 使用簡易計算機　←請出題老師勾選，謝謝！

命題老師：(Teacher)	(簽章) 103年 9月 15日 (Signature & date)	試題隨卷繳交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。

Remarks : For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.