| 考試科目
Course | 組合學 | 開課系級
Dept, &
Class | 研究所 | 日　　期
Date,
Period | 104 年 9 月 21 日
下午 1:00~4:00 | 試題編號
Course
No. | |

本試卷共有 6 個題目，

碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。

博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Show that there are n^{n-2} labeled trees of n vertices labeled with 1,2,...,n.

2. Given a convex n-gon such that no three diagonals meet at the same point inside the n-gon, how many line segments are the diagonals divided by their intersections?

3. How many regions can be formed when n mutually intersecting planes are drawn in a three-dimensional space such that no four planes intersect at a common point and no two planes have parallel intersection lines in a third plane?

4. Let $A_1, A_2, ..., A_n$ be n sets in the universe U and S_k denote the sum of the sizes of all k-tuple intersections of the $A_i's$. Show that the number of elements in exactly m sets is $\sum_{i=0}^{n-m}(-1)^i C_m^{m+i} S_{m+i}$.

5. Show that $\sqrt{1+x} = \sum_{k \geq 0} C_k^{\frac{1}{2}} x^k$,

   where $C_k^{\frac{1}{2}} = \begin{cases} 1 & if\ k = 0, \\ \dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)...\left(\frac{1}{2}-k+1\right)}{k!} & if\ k \geq 1. \end{cases}$

   (Do not use the methods from Calculus!)

6. How many ways are there to color the faces of a cube using n colors?

本考試：☑ 不需使用簡易計算機，☐ 使用簡易計算機　　　←請出題老師勾選，謝謝！

簽章）104 年 9 月 14 日
Signature & date ）

試 題 隨 卷 繳 交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（ 紅色不能製版請勿使用 ）。

Remarks ：For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.