

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

系別	應用數學系	考試 科目	組合學	考試 日期	2022 年 2 月 21 日	考試 時間	09:00 至 12:00
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注意事項

- 務必作答於答案卷並標明題號，請勿作答於試題卷上，否則不予計分。
- 本試題卷共有 4 個問題，總計 100 分。

- (20 %) (a) Draw three different labeled trees of three vertices.
(b) How many different labeled trees of n vertices?
(c) Prove your answer in (b).
- (25 %) (a) State the Hall's marriage theorem for bipartite graphs.
(b) Prove the statement you have written in (a).
(c) Let G be a bipartite graph with bipartition $X \cup Y$ such that each vertex $x \in X$ has degree k and each vertex $y \in Y$ has degree at most k . Show that G has k families of disjoint complete matchings from X to Y .
- (40 %) Let $[n]$ denote the set $\{1, 2, \dots, n\}$. Determine the following numbers.
 - $$\sum_{A \subseteq [50]} |A|;$$
 - $$|\{(A_1, A_2) : A_i \subseteq [50], A_1 \cap A_2 = \emptyset\}|;$$
 - $$|\{(A_1, A_2, A_3, A_4) : A_i \subseteq [50], A_1 \cup A_2 \cup A_3 \cup A_4 = [50]\}|;$$
 - The number of binary circular sequences of length 9.
- (15 %) (a) State the Fisher's inequality for a 2 -(v, k, λ) design;
(b) Prove your statement in (a).

命題老師簽章：

(Teacher's Signature)

日期：

(Date) 2022 年 2 月 9 日

■ 試題隨卷繳交

■ 不可使用計算機

命題紙使用說明：

Remarks :

請使用黑色墨水正楷書寫或打字(紅色不能製版請勿使用)。

For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.