

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

系別	應用數學系	考試 科目	數值方法	考試 日期	2026 年 3 月 2 日	考試 時間	13:00-16:00
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3. (20 %) Let $v \in \mathbb{R}^n$ be a unit vector, i.e., $\|v\| = 1$. The Householder transformation is defined as

$$Q = I - 2vv^\top$$

where I is the $n \times n$ identity matrix and the operator \top denotes the transpose.

(a) (6%) Show that $Q = Q^\top = Q^{-1}$.

(b) (8%) Find all the eigenvalues and the associated eigenvectors of the matrix Q .

(c) (6%) Let $x = \begin{pmatrix} 3 & 4 & 0 & \dots & 0 \end{pmatrix}^\top$. Find the Householder transformation Q such that

$$Qx = \begin{pmatrix} 5 & 0 & 0 & \dots & 0 \end{pmatrix}^\top.$$

4. (12 %) Construct the natural cubic spline that interpolates the data $(1, 2)$, $(2, 3)$, and $(3, 5)$.

5. (12 %) Derive the Adams-Bashforth two-step formula

$$x_{n+1} = x_n + \frac{h}{2} [3f_n - f_{n-1}]$$

and determine its order.

6. (16 %) Suppose that $f : [a, b] \rightarrow [a, b]$ is continuous on interval $[a, b]$ and is a contraction, i.e., there exists a constant $L \in (0, 1)$ such that

$$|f(x) - f(y)| \leq L|x - y|, \quad \text{for all } x, y \in [a, b].$$

Prove that there exists a unique fixed point in $[a, b]$ and that the fixed point iteration $x_{n+1} = f(x_n)$ converges to the fixed point for any $x_0 \in [a, b]$.

命題老師簽章：

(Teacher's Signature)

日期：

(Date)

年 月 日

■ 試題隨卷繳交

■ 不可使用計算機

命題紙使用說明： 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。

Remarks : For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.