國立政治大學 **112** 學年度第二學期 博士班資格考 試題卷
NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

系別	應用數學系	考試科目	實變函數論	考試日期	**2024 年 2 月 26 日**	考試時間	**09:00-12:00**

注意事項
- 務必作答於答案卷並標明題號，請勿作答於試題卷上，否則不予計分。
- 本試題卷共有 **7** 個問題**,** 總計 **100** 分。

1. (15 %) Let $\{f_n\}$ be a sequence of continuous functions defined on $\mathbb{R}$. Show that the set C of points where this sequence converges is an $F_{\sigma\delta}$.

2. (15 %) Let $m^*(A)$ be Lebesgue outer measure of $A \subset \mathbb{R}$. Show that, if A is any set with $m^*(A) > 0$, then there is a non-measurable set $E \subset A$.

3. (15 %) If $f \in L(0,1)$, show that $x^k f(x) \in L(0,1)$ for $k = 1, 2, \cdots$, and $\displaystyle\lim_{k\to\infty} \int_0^1 x^k f(x) = 0$.

4. (15 %) Let f be nonnegative and measurable on $\mathbb{R}$ and $0 < p < \infty$. Prove that

$$\int_{\mathbb{R}} f(x)^p dx = p \int_0^\infty t^{p-1} |\{x \in \mathbb{R} : f(x) > t\}| dt$$

by Fubini theorem.

5. (15 %) Prove or disprove that if f is of bounded variation and continuous on $[a,b]$, then f is absolutely continuous on $[a,b]$.

6. (15 %) Let C_0^∞ be the class of infinitely differentiable functions with compact support. Show that C_0^∞ is dense in $L^p(\mathbb{R}^n)$ for $1 \le p < \infty$.

7. (10 %) Let

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt, \qquad \text{for } 0 < x < \infty.$$

Show that $\log \Gamma(x)$ is a convex function on $(0, \infty)$ by Hölder's inequality.

命題老師簽章： (Teacher's Signature)	日期： (Date) 年 月 日	■ 試題隨卷繳交 ■ 不可使用計算機

命題紙使用說明： 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字 (紅色不能製版請勿使用)。
Remarks： For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.