國立政治大學應用數學系九十三學年度第一學期研究生學科考試試題

科目：實變函數論

1. Let (X, β) be a measurable space, $\{\mu_n\}$ a sequence of measures which converge setwise to a measure μ, and $\{f_n\}$ a sequence of nonnegative measurable functions which converge pointwise to the function f. Show that
$$\int f\,d\mu \le \underline{\lim} \int f_n\,d\mu_n.$$

2. Let (X, β, μ) be a measure space and g a nonnegative measurable function in X. Set
$$\nu E = \int_E g\,d\mu.$$
Prove that ν is a measure in β.

3. If (X, β, μ) be a measure space, $E_i \in \beta$ and $E_i \supset E_{i+1}$, then
$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n.$$

4. Suppose that f is a bounded linear functional in Hilbert space H. Then there exists $y \in H$ such that
$$f(x) =< x, y >,\ \forall x \in H.$$

5. If a linear operator is continuous at one point, it is bounded.

6. If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in metric space (X, ρ), then $\rho(x_n, y_n)$ converges.

7. Suppose that $f : [0,1] \to [0,1]$ is continuous, then there exists $x \in [0,1]$ such that
$$f(x) = x.$$