國立政治大學 **109** 學年度第一學期 博士班資格考 試題卷

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

系別	應用數學系	考試科目	實變函數論	考試日期	**2020** 年 **9** 月 **21** 日	考試時間	**09:00** 至 **12:00**

注意事項

- 務必作答於答案卷並標明題號，請勿作答於試題卷上，否則不予計分。
- 本試題卷共有 **7** 個問題**,** 總計 **100** 分。

1. (15 %) Let $f(x)$ be a real valued continuous function on $[0,1]$. Find the limit
$$\lim_{n\to\infty}(n+1)\int_0^1 x^n f(x)dx.$$

2. (15 %) Given a measurable set $A \subset [0,1]$ with $|A| > 0$. Let $B = \cos(A) = \{cos(x), x \in A\}$. Show that the measure of B is strictly less than the measure of A, i.e $|B| < |A|$.

3. (15 %) Let $\{\phi_k\}$ be an orthonormal system and $\{c_k\}$ be a sequence of numbers in ℓ^2. Show that there exists a function $f \in L^2$ such that its Fouerier series with respect to $\{\phi_k\}$ is exactly equal to $\sum c_k\phi_k(x)$.

4. (15 %) Given a measure space $(E, \sum, \mu)$ with $\mu(E) > 0$ and a bounded non-constant measurable function $f(x)$. Show that there exists $\lambda \in \mathbb{R}$ such that $\mu(\{f \le \lambda\}) > 0$ and $\mu(\{f > \lambda\}) > 0$.

5. (10 %) Let $\{f_k\}$ be a sequence of measurable functions on a measurable set E. Assume that f is a function on E, and let $E_k = \{x \in E : |f(x) - f_k(x)| > \frac{1}{k^2}\}$. Suppose $|E_k|_e < \frac{1}{k^2}$, $\forall k$. Show that f is a measurable function.

6. (15 %) Let $f : S \to \mathbb{R}$ be a uniformly continuous function defined on a subset S of a metric space M. Show that (1) f extends to a uniformly continuous function $\overline{f}$ on $\overline{S}$. (2) Such an extension is unique on $\overline{S}$.

7. (15 %) Let X be a normed space. We say $\{x_i\}_{i=1}^N \subset X$ is linearly independent if $\alpha_1x_1 + \alpha_2x_2 + \cdots + \alpha_Nx_N = 0$ if and only if $\alpha_i = 0$ for all i. Now assuming $\{x_i\}_{i=1}^N$ is linearly independent. Prove that there is a constant $c > 0$ such that
$$||\lambda_1x_1 + \cdots \lambda_Nx_N|| \ge c(|\lambda_1| + \cdots |\lambda_N|),$$
for any choices of scalars $\lambda_1, \cdots, \lambda_N$.

命題老師簽章： **(Teacher's Signature)**	日期： **(Date)**	年 月 日	■ 試題隨卷繳交 ■ 不可使用計算機

命題紙使用說明： 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字 (紅色不能製版請勿使用)。
Remarks ： For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.