國立政治大學 **113** 學年度第一學期 博士班資格考 試題卷

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

系別	應用數學系	考試科目	實變函數論	考試日期	**2024** 年 **9** 月 **16** 日	考試時間	**09:00-12:00**

注意事項

- 務必作答於答案卷並標明題號，請勿作答於試題卷上，否則不予計分。
- 本試題卷共有 **7** 個問題**,** 總計 **100** 分。

1. (15 %) Suppose $f_k, f \in L^p$ such that $f_k \to f$ a.e., $1 < p < \infty$. If $\|f_k\|_p \leq M < \infty$, show that $\int f_k g \, d\mu \to \int fg \, d\mu$ for all $g \in L^q$, where $1/p + 1/q = 1$.

2. (15 %) Let $(X, \mathcal{B}, \mu)$ be a probability space. Suppose $\int_A f \, d\mu = 0$ for every $A \in \mathcal{B}$. Show that $f = 0$ a.e.

3. (15 %) Let $(X, \mathcal{B}, \mu)$ be a probability space and $\mu(T^{-1}A) = \mu(A)$ for all $A \in \mathcal{B}$. Suppose

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mu(T^{-i}A \cap B) = \mu(A)\mu(B) \qquad \forall\, A, B \in \mathcal{B}.$$

   Show that $\mu(E) = 0$ or $\mu(E) = 1$ provided $\mu(T^{-1}(E)\Delta E) = 0$, where Δ is the symmetric difference.

4. (15 %) Show that there exist measurable functions $f$and g such that $f \circ g$ is not measurable.

5. (15 %) Let $\mathcal{H}$ be a Hilbert space. Let $f_n, f \in \mathcal{H}$ for $n \in \mathbb{N}$ such that $\langle f_n, g\rangle \to \langle f, g\rangle$ as $n \to \infty$ for all $g \in \mathcal{H}$. Then $\{f_n\}$ is bounded.

6. (15 %) Let $p \in \mathbb{N}$ and X be a measurable space. Show that $L^p(X)$ is a Banach space with respect to L^p-norm.

7. (10 %) Let $(X, \mathcal{B}, \mu)$ be a probability space and $\mu(T^{-1}A) = \mu(A)$ for all $A \in \mathcal{B}$. Suppose $A \in \mathcal{B}$ is of positive measure. Show that for almost every $x \in A$, there exists $k \in \mathbb{N}$ such that $T^k(x) \in A$.

命題老師簽章： **(Teacher's Signature)**	日期： **(Date)**	年 月 日	■ 試題隨卷繳交 ■ 不可使用計算機

命題紙使用說明： 試題將用原件印製，敬請使用黑色墨水正楷書寫或打字 (紅色不能製版請勿使用)。
Remarks : For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.