國立政治大學 應用數學系 100 學年度第 一 學期　學科 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

考試科目 Course	數理統計	開課系級 Dept, & Class	研究所	日期 Date, Period	100 年 9 月 19 日 上午 9:00~12:00	試題編號 Course No.	

本試卷共有 6 個題目，
碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。
博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

To earn your credits, you must show your work.

You don't have to use the calculator. However, you may use the information at the end of page 2 and the attached statistical table.

1. Let X_1, X_2, X_3 be independent with X_i having density $f(x_i) = \exp(-x_i)$, $x_i>0$. Let $U_1=X_1+X_2+X_3$, $U_2=X_2/U_1$, and $U_3=X_3/U_1$. (a)Find the joint density of U_1, U_2, and U_3. (b) Find the marginal density of U_1. (c) Find the conditional density of U_1 given U_2 and U_3.

2. A prisoner is in a cell with four doors. He chooses a door at random (each with probability 1/4). The first door leads to a tunnel which leads to freedom in one day. The second door leads to a long tunnel which leads to freedom in three days. The third tunnel is a trap which leads back to the cell in two days, and the forth tunnel is also a trap which leads back to the prison cell, but in five days. Each time the prisoner gets back to the cell, he chooses a door at random from the four doors (again, each with probability 1/4). (That is, he does not remember the door he chose the previous time.) Find the expected time till the prisoner escapes.

本考試：☑ 不需使用簡易計算機，□ 使用簡易計算機　　←請出題老師勾選，謝謝！

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博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

3. Suppose we observe X_i , i=1, …, n, independent, with $X_i \sim E(i\theta)$, $\theta > 0$, that is, $f(x_i;\theta) = (i\theta)^{-1} \exp(\frac{-x_i}{i\theta})$, θ>0. (a) Find the MLE $T_n(X_1, X_2, \ldots, X_n)$ of θ. (b) Find the asymptotical distribution of T_n as n goes to infinity.

4. State and prove the Neyman-Pearson Theorem.

5. Let $X_1, X_2, \ldots, X_n$ be independent, and let X_i be normal distribution with mean $i\theta$ and variance 1. Find the uniformly most powerful (UMP) size-0.025 test that $\theta = 3$ against $\theta < 3$ when n=3.

6. Let X_1 and X_2 be two independent and identical Bernoulli distributions with mean θ. That is, $X_i \sim B(1, \theta)$ for i=1 and 2. Consider testing θ=0.5 against θ>0.5. Let $\Phi(X_1, X_2)$ be the nonrandomized test which rejects the null hypothesis if $X_1=1$. (a) Show that $T=X_1+X_2$ is a sufficient statistic. (b) Find $\Phi^*(T)=E\langle\Phi(X_1,X_2)|T\rangle$. (c) Which test ($\Phi(X_1,X_2)$ or $\Phi^*(T)$) has more power? Why?

Note:

x	6	7	8	10	11	12	13	14	15
$\sqrt{x}$	2.45	2.65	2.83	3.16	3.32	3.46	3.61	3.74	3.87

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