國立政治大學 應用數學系 101 學年度第二學期　學科 考試試題

NATIONAL CHENGCHI UNIVERSITY EXAMINATION FORM

| 考試科目 Course | 數理統計 | 開課系級 Dept, & Class | 研究所 | 日　　期 Date, Period | 102 年 3 月 4 日 上午 9:00~12:00 | 試題編號 Course No. | |

本試卷共有 6 個題目，

碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。

博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

To earn your credits, you must show your work.

You don't have to use the calculator. However, you may use the information at the end of page 2.

1. Let $X_1, X_2, \ldots, X_n$ be a sample from a continuous uniform on the interval $[0, \theta]$.

   (a) Find a sufficient statistic for θ.

   (b) Find a pivotal quantity for θ based on a sufficient statistics for θ.

   (c) Find a $(1-\alpha)$ confidence interval for θ, based on the pivotal quantity in (b).

   (d) Is your confidence interval in (c) the shortest-length confidence interval for θ? If not, find it.

2. Suppose we have a population of N objects, M of which are labeled successes and the remaining N-M failures. We draw a sample of size n without replacement from this population. Let X be the number of successes in the sample. Find the uniformly most powerful size α test that $M \leq M_0$ against $M > M_0$.

3. Let $X_1, X_2, \ldots, X_n$ be independent, with $X_i \sim P(i\theta)$, $\theta > 0$.

   (a) Find the MLE of $\exp(-2\theta)$.

   (b) Find the best unbiased estimator of $\exp(-2\theta)$.

4. Prove or disprove the following two statements.

   (a) If $X_n \overset{p}{\to} a$ and g(x) is continuous at x=a, then $g(X_n) \overset{p}{\to} g(a)$.

   (b) If $X_n \overset{p}{\to} a$, $Y_n \overset{p}{\to} b$, and g(x, y) is continuous at $(x, y) = (a, b)$, then $g(X_n, Y_n) \overset{p}{\to} g(a, b)$.

本考試：☑不需使用簡易計算機，□ 使用簡易計算機　　　　←請出題老師勾選，謝謝！

命 題 老 師：
(T e a c h e r)

試 題 隨 卷 繳 交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（ 紅色不能製版請勿使用 ）。

Remarks ： For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.

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5. A company has three machines A, B, and C for making 1 kΩ resistors. It has been observed that 80% of resisters produced by A are within 50 Ω of the nominal value. Machine B produces 90 % of resistors within 50 Ω of the nominal value. The percentage for machine C is 60%. Each minute, machine A produce 300 resistors, B produces 400 resistors, and C produces 300 resistors. All of the resistors are mixed together at random in one bin and packed for shipment.

(a) What is the probability that the company ships a resistor that is within 50 Ω of the nominal value?

(b) What is the probability that an acceptable resistor comes from machine C.

6. Random variables X and Y have the joint density function

$$f(x, y) = c \exp -[2x^2 - 4xy + 4y^2]$$

(a) What are E[X] and E[Y]?

(b) Find the correlation coefficient of X and Y.

(c) What are Var[X] and var[Y]?

(d) What is the constant c?

(e) Are X and Y independent?

Note:

If a random variable X has a Poisson distribution with mean m>0, then X has the

density function $f(x) = \dfrac{e^{-m}m^x}{x!}$, x=0, 1, 2, …

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命 題 老 師：
(Teacher)

試 題 隨 卷 繳 交

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