國立政治大學 應用數學系 105 學年度第一學期　學科 考試試題

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| 考試科目 Course | 數理統計 | 開課系級 Dept. & Class | 研究所 | 日期 Date, Period | 105 年 9 月 19 日 上午 9:00~12:00 | 試題編號 Course No. | |
|---|---|---|---|---|---|---|---|

本試卷共有 6 個題目，
碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。
博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. Let $(X, Y)$ be a random point chosen uniformly on the region $R = \{(x,y) \mid |x| + |y| \le 1\}$.
   (1) Find the conditional density of $Y$, given $X = -\frac{1}{2}$.
   (2) Find $Var(Y \mid X = -\frac{1}{2})$.

2. Show that
   (1) $E(Y) = E\,E(Y \mid X)$
   (2) $Var(Y) = E\,Var(Y \mid X) + Var\,E(Y \mid X)$

3. (1) Phone calls are received at a certain residence as a Poisson process with parameter $\lambda = 2$ per hour. How long can her shower be if she wishes the probability of receiving no phone calls to be at most 0.5?
   (2) Let $X = (X_1, X_2, X_3)$ have joint moment-generating function $M(t_1, t_2, t_3) = (1 - t_1 - 2t_2)^{-4}(1 - t_1 + 3t_3)^{-2}(1 - t_2)^{-3}$. Find $Cov(X_2, X_3)$.

4. Let $X_1, \cdots, X_n \overset{iid}{\sim} f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$.
   (1) Find the best unbiased estimator of $\theta$.

本考試：■ 不需使用簡易計算機，□ 使用簡易計算機　←請出題老師勾選，謝謝！

| 命題老師：(Teacher) | （簽章）105 年 9 月 12 日 ( Signature & date ) | 試題隨卷繳交 |
|---|---|---|

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博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

(2) Is the best unbiased estimator efficient ?

5. (1) Let $(X, Y)$ have a trinomial distribution with $n=2$ and parameters $(\theta_1, \theta_2)$. Find the most powerful size-0.18 test of the null hypothesis that $\theta_1 = \theta_2 = 0.4$ against $\theta_1 = 0.2$ and $\theta_2 = 0.6$.

(2) Let $X_1, \cdots, X_n$ be independent, with $X_i \sim N(\theta, 1)$. Find the size-$\alpha$ LRT for testing that $\theta = 0$ against $\theta \neq 0$.

6. (1) Let $X_1, X_2, X_3$ be independent, with $X_i \sim P(i\theta)$. Find the UMP size-0.05 test that $\theta = 3/2$ against $\theta < 3/2$.

(2) Let $X_1, X_2, X_3$ be independent and $X_i \sim E(i\theta)$, $\theta > 0$. Find the UMP size-0.05 test that $\theta \geq 2$ against $\theta < 2$.

本考試：■ 不需使用簡易計算機，□ 使用簡易計算機　←請出題老師勾選，謝謝！

| 命題老師: (Teacher) | ( 簽章 ) 105 年 9 月 12 日 ( Signature & date ) | 試題隨卷繳交 |
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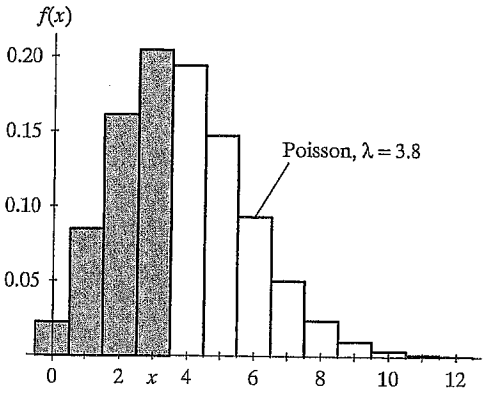
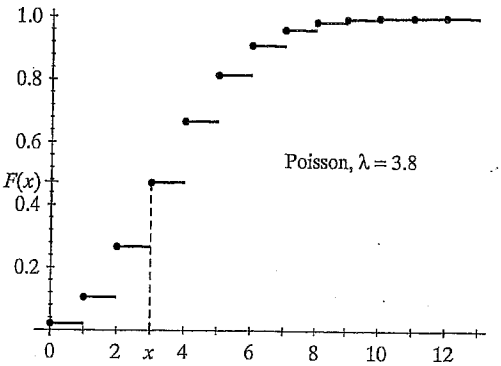
## TABLE III: The Poisson Distribution




$$F(x) = P(X \le x) = \sum_{k=0}^{x} \frac{\lambda^k e^{-\lambda}}{k!}$$

| | $\lambda = E(X)$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 0 | 0.905 | 0.819 | 0.741 | 0.670 | 0.607 | 0.549 | 0.497 | 0.449 | 0.407 | 0.368 |
| 1 | 0.995 | 0.982 | 0.963 | 0.938 | 0.910 | 0.878 | 0.844 | 0.809 | 0.772 | 0.736 |
| 2 | 1.000 | 0.999 | 0.996 | 0.992 | 0.986 | 0.977 | 0.966 | 0.953 | 0.937 | 0.920 |
| 3 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.994 | 0.991 | 0.987 | 0.981 |
| 4 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.996 |
| 5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 |
| 6 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| $x$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.333 | 0.301 | 0.273 | 0.247 | 0.223 | 0.202 | 0.183 | 0.165 | 0.150 | 0.135 |
| 1 | 0.699 | 0.663 | 0.627 | 0.592 | 0.558 | 0.525 | 0.493 | 0.463 | 0.434 | 0.406 |
| 2 | 0.900 | 0.879 | 0.857 | 0.833 | 0.809 | 0.783 | 0.757 | 0.731 | 0.704 | 0.677 |
| 3 | 0.974 | 0.966 | 0.957 | 0.946 | 0.934 | 0.921 | 0.907 | 0.891 | 0.875 | 0.857 |
| 4 | 0.995 | 0.992 | 0.989 | 0.986 | 0.981 | 0.976 | 0.970 | 0.964 | 0.956 | 0.947 |
| 5 | 0.999 | 0.998 | 0.998 | 0.997 | 0.996 | 0.994 | 0.992 | 0.990 | 0.987 | 0.983 |
| 6 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.998 | 0.997 | 0.997 | 0.995 |
| 7 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 |
| 8 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| $x$ | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 | 3.2 | 3.4 | 3.6 | 3.8 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.111 | 0.091 | 0.074 | 0.061 | 0.050 | 0.041 | 0.033 | 0.027 | 0.022 | 0.018 |
| 1 | 0.355 | 0.308 | 0.267 | 0.231 | 0.199 | 0.171 | 0.147 | 0.126 | 0.107 | 0.092 |
| 2 | 0.623 | 0.570 | 0.518 | 0.469 | 0.423 | 0.380 | 0.340 | 0.303 | 0.269 | 0.238 |
| 3 | 0.819 | 0.779 | 0.736 | 0.692 | 0.647 | 0.603 | 0.558 | 0.515 | 0.473 | 0.433 |
| 4 | 0.928 | 0.904 | 0.877 | 0.848 | 0.815 | 0.781 | 0.744 | 0.706 | 0.668 | 0.629 |
| 5 | 0.975 | 0.964 | 0.951 | 0.935 | 0.916 | 0.895 | 0.871 | 0.844 | 0.816 | 0.785 |
| 6 | 0.993 | 0.988 | 0.983 | 0.976 | 0.966 | 0.955 | 0.942 | 0.927 | 0.909 | 0.889 |
| 7 | 0.998 | 0.997 | 0.995 | 0.992 | 0.988 | 0.983 | 0.977 | 0.969 | 0.960 | 0.949 |
| 8 | 1.000 | 0.999 | 0.999 | 0.998 | 0.996 | 0.994 | 0.992 | 0.988 | 0.984 | 0.979 |
| 9 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 | 0.994 | 0.992 |
| 10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 |
| 11 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 12 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

**TABLE III:** *continued*

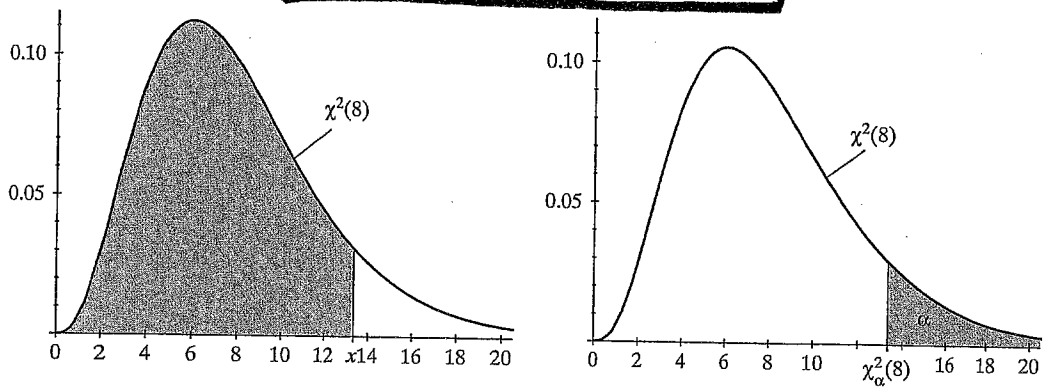
| $x$ | 4.2 | 4.4 | 4.6 | 4.8 | 5.0 | 5.2 | 5.4 | 5.6 | 5.8 | 6.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.015 | 0.012 | 0.010 | 0.008 | 0.007 | 0.006 | 0.005 | 0.004 | 0.003 | 0.002 |
| 1 | 0.078 | 0.066 | 0.056 | 0.048 | 0.040 | 0.034 | 0.029 | 0.024 | 0.021 | 0.017 |
| 2 | 0.210 | 0.185 | 0.163 | 0.143 | 0.125 | 0.109 | 0.095 | 0.082 | 0.072 | 0.062 |
| 3 | 0.395 | 0.359 | 0.326 | 0.294 | 0.265 | 0.238 | 0.213 | 0.191 | 0.170 | 0.151 |
| 4 | 0.590 | 0.551 | 0.513 | 0.476 | 0.440 | 0.406 | 0.373 | 0.342 | 0.313 | 0.285 |
| 5 | 0.753 | 0.720 | 0.686 | 0.651 | 0.616 | 0.581 | 0.546 | 0.512 | 0.478 | 0.446 |
| 6 | 0.867 | 0.844 | 0.818 | 0.791 | 0.762 | 0.732 | 0.702 | 0.670 | 0.638 | 0.606 |
| 7 | 0.936 | 0.921 | 0.905 | 0.887 | 0.867 | 0.845 | 0.822 | 0.797 | 0.771 | 0.744 |
| 8 | 0.972 | 0.964 | 0.955 | 0.944 | 0.932 | 0.918 | 0.903 | 0.886 | 0.867 | 0.847 |
| 9 | 0.989 | 0.985 | 0.980 | 0.975 | 0.968 | 0.960 | 0.951 | 0.941 | 0.929 | 0.916 |
| 10 | 0.996 | 0.994 | 0.992 | 0.990 | 0.986 | 0.982 | 0.977 | 0.972 | 0.965 | 0.957 |
| 11 | 0.999 | 0.998 | 0.997 | 0.996 | 0.995 | 0.993 | 0.990 | 0.988 | 0.984 | 0.980 |
| 12 | 1.000 | 0.999 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 | 0.995 | 0.993 | 0.991 |
| 13 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 |
| 14 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 |
| 15 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

| $x$ | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 | 10.5 | 11.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 0.011 | 0.007 | 0.005 | 0.003 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 |
| 2 | 0.043 | 0.030 | 0.020 | 0.014 | 0.009 | 0.006 | 0.004 | 0.003 | 0.002 | 0.001 |
| 3 | 0.112 | 0.082 | 0.059 | 0.042 | 0.030 | 0.021 | 0.015 | 0.010 | 0.007 | 0.005 |
| 4 | 0.224 | 0.173 | 0.132 | 0.100 | 0.074 | 0.055 | 0.040 | 0.029 | 0.021 | 0.015 |
| 5 | 0.369 | 0.301 | 0.241 | 0.191 | 0.150 | 0.116 | 0.089 | 0.067 | 0.050 | 0.038 |
| 6 | 0.527 | 0.450 | 0.378 | 0.313 | 0.256 | 0.207 | 0.165 | 0.130 | 0.102 | 0.079 |
| 7 | 0.673 | 0.599 | 0.525 | 0.453 | 0.386 | 0.324 | 0.269 | 0.220 | 0.179 | 0.143 |
| 8 | 0.792 | 0.729 | 0.662 | 0.593 | 0.523 | 0.456 | 0.392 | 0.333 | 0.279 | 0.232 |
| 9 | 0.877 | 0.830 | 0.776 | 0.717 | 0.653 | 0.587 | 0.522 | 0.458 | 0.397 | 0.341 |
| 10 | 0.933 | 0.901 | 0.862 | 0.816 | 0.763 | 0.706 | 0.645 | 0.583 | 0.521 | 0.460 |
| 11 | 0.966 | 0.947 | 0.921 | 0.888 | 0.849 | 0.803 | 0.752 | 0.697 | 0.639 | 0.579 |
| 12 | 0.984 | 0.973 | 0.957 | 0.936 | 0.909 | 0.876 | 0.836 | 0.792 | 0.742 | 0.689 |
| 13 | 0.993 | 0.987 | 0.978 | 0.966 | 0.949 | 0.926 | 0.898 | 0.864 | 0.825 | 0.781 |
| 14 | 0.997 | 0.994 | 0.990 | 0.983 | 0.973 | 0.959 | 0.940 | 0.917 | 0.888 | 0.854 |
| 15 | 0.999 | 0.998 | 0.995 | 0.992 | 0.986 | 0.978 | 0.967 | 0.951 | 0.932 | 0.907 |
| 16 | 1.000 | 0.999 | 0.998 | 0.996 | 0.993 | 0.989 | 0.982 | 0.973 | 0.960 | 0.944 |
| 17 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.995 | 0.991 | 0.986 | 0.978 | 0.968 |
| 18 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.096 | 0.993 | 0.988 | 0.982 |
| 19 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.994 | 0.991 |
| 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.995 |
| 21 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 |
| 22 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 23 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

**TABLE III:** *continued*

| $x$ | 11.5 | 12.0 | 12.5 | 13.0 | 13.5 | 14.0 | 14.5 | 15.0 | 15.5 | 16.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.011 | 0.008 | 0.005 | 0.004 | 0.003 | 0.002 | 0.001 | 0.001 | 0.001 | 0.000 |
| 5 | 0.028 | 0.020 | 0.015 | 0.011 | 0.008 | 0.006 | 0.004 | 0.003 | 0.002 | 0.001 |
| 6 | 0.060 | 0.046 | 0.035 | 0.026 | 0.019 | 0.014 | 0.010 | 0.008 | 0.006 | 0.004 |
| 7 | 0.114 | 0.090 | 0.070 | 0.054 | 0.041 | 0.032 | 0.024 | 0.018 | 0.013 | 0.010 |
| 8 | 0.191 | 0.155 | 0.125 | 0.100 | 0.079 | 0.062 | 0.048 | 0.037 | 0.029 | 0.022 |
| 9 | 0.289 | 0.242 | 0.201 | 0.166 | 0.135 | 0.109 | 0.088 | 0.070 | 0.055 | 0.043 |
| 10 | 0.402 | 0.347 | 0.297 | 0.252 | 0.211 | 0.176 | 0.145 | 0.118 | 0.096 | 0.077 |
| 11 | 0.520 | 0.462 | 0.406 | 0.353 | 0.304 | 0.260 | 0.220 | 0.185 | 0.154 | 0.127 |
| 12 | 0.633 | 0.576 | 0.519 | 0.463 | 0.409 | 0.358 | 0.311 | 0.268 | 0.228 | 0.193 |
| 13 | 0.733 | 0.682 | 0.629 | 0.573 | 0.518 | 0.464 | 0.413 | 0.363 | 0.317 | 0.275 |
| 14 | 0.815 | 0.772 | 0.725 | 0.675 | 0.623 | 0.570 | 0.518 | 0.466 | 0.415 | 0.368 |
| 15 | 0.878 | 0.844 | 0.806 | 0.764 | 0.718 | 0.669 | 0.619 | 0.568 | 0.517 | 0.467 |
| 16 | 0.924 | 0.899 | 0.869 | 0.835 | 0.798 | 0.756 | 0.711 | 0.664 | 0.615 | 0.566 |
| 17 | 0.954 | 0.937 | 0.916 | 0.890 | 0.861 | 0.827 | 0.790 | 0.749 | 0.705 | 0.659 |
| 18 | 0.974 | 0.963 | 0.948 | 0.930 | 0.908 | 0.883 | 0.853 | 0.819 | 0.782 | 0.742 |
| 19 | 0.986 | 0.979 | 0.969 | 0.957 | 0.942 | 0.923 | 0.901 | 0.875 | 0.846 | 0.812 |
| 20 | 0.992 | 0.988 | 0.983 | 0.975 | 0.965 | 0.952 | 0.936 | 0.917 | 0.894 | 0.868 |
| 21 | 0.996 | 0.994 | 0.991 | 0.986 | 0.980 | 0.971 | 0.960 | 0.947 | 0.930 | 0.911 |
| 22 | 0.999 | 0.997 | 0.995 | 0.992 | 0.989 | 0.983 | 0.976 | 0.967 | 0.956 | 0.942 |
| 23 | 0.999 | 0.999 | 0.998 | 0.996 | 0.994 | 0.991 | 0.986 | 0.981 | 0.973 | 0.963 |
| 24 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.995 | 0.992 | 0.989 | 0.984 | 0.978 |
| 25 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 | 0.994 | 0.991 | 0.987 |
| 26 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.995 | 0.993 |
| 27 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 |
| 28 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.998 |
| 29 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 |
| 31 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 32 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 33 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 34 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 35 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

## TABLE IV: The Chi-Square Distribution




$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

| | $P(X \le x)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.010 | 0.025 | 0.050 | 0.100 | 0.900 | 0.950 | 0.975 | 0.990 |
| $r$ | $\chi^2_{0.99}(r)$ | $\chi^2_{0.975}(r)$ | $\chi^2_{0.95}(r)$ | $\chi^2_{0.90}(r)$ | $\chi^2_{0.10}(r)$ | $\chi^2_{0.05}(r)$ | $\chi^2_{0.025}(r)$ | $\chi^2_{0.01}(r)$ |
| 1 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 |
| 2 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 |
| 3 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.34 |
| 4 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.14 | 13.28 |
| 5 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.07 | 12.83 | 15.09 |
| 6 | 0.872 | 1.237 | 1.635 | 2.204 | 10.64 | 12.59 | 14.45 | 16.81 |
| 7 | 1.239 | 1.690 | 2.167 | 2.833 | 12.02 | 14.07 | 16.01 | 18.48 |
| 8 | 1.646 | 2.180 | 2.733 | 3.490 | 13.36 | 15.51 | 17.54 | 20.09 |
| 9 | 2.088 | 2.700 | 3.325 | 4.168 | 14.68 | 16.92 | 19.02 | 21.67 |
| 10 | 2.558 | 3.247 | 3.940 | 4.865 | 15.99 | 18.31 | 20.48 | 23.21 |
| 11 | 3.053 | 3.816 | 4.575 | 5.578 | 17.28 | 19.68 | 21.92 | 24.72 |
| 12 | 3.571 | 4.404 | 5.226 | 6.304 | 18.55 | 21.03 | 23.34 | 26.22 |
| 13 | 4.107 | 5.009 | 5.892 | 7.042 | 19.81 | 22.36 | 24.74 | 27.69 |
| 14 | 4.660 | 5.629 | 6.571 | 7.790 | 21.06 | 23.68 | 26.12 | 29.14 |
| 15 | 5.229 | 6.262 | 7.261 | 8.547 | 22.31 | 25.00 | 27.49 | 30.58 |
| 16 | 5.812 | 6.908 | 7.962 | 9.312 | 23.54 | 26.30 | 28.84 | 32.00 |
| 17 | 6.408 | 7.564 | 8.672 | 10.08 | 24.77 | 27.59 | 30.19 | 33.41 |
| 18 | 7.015 | 8.231 | 9.390 | 10.86 | 25.99 | 28.87 | 31.53 | 34.80 |
| 19 | 7.633 | 8.907 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 |
| 20 | 8.260 | 9.591 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 |
| 21 | 8.897 | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 |
| 22 | 9.542 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 |
| 23 | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 |
| 24 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 |
| 25 | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 |
| 26 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.88 | 41.92 | 45.64 |
| 27 | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 |
| 28 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 |
| 29 | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 |
| 30 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 |
| 40 | 22.16 | 24.43 | 26.51 | 29.05 | 51.80 | 55.76 | 59.34 | 63.69 |
| 50 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 |
| 60 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 |
| 70 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 |
| 80 | 53.34 | 57.15 | 60.39 | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 |

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E.S.Pearson and H.O.Hartley.