

國立政治大學應用數學系九十二學年度第一學期研究生學科考試試題

科目：數理統計

請任選五題作答：

1. 試解釋下列各名詞：

(a) power function (b) p-value (c) unbiased test (d) sufficient statistic

2. Let X_i be independent and $X_i \sim \text{Poisson}(i\theta)$, $\theta > 0$, $i = 1, 2, \dots, n$. Find the best unbiased estimator of θ .

3. Let X_1, X_2, \dots, X_n be independently uniformly distributed on $(0, \theta)$. Does there exist the best unbiased estimator of θ ? If yes, find it.

4. State and prove the Neyman-Pearson Theorem on testing simple hypothesis verse another simple hypothesis.

5. Let X be a discrete random variable with density $f(x; \theta)$ giving in the following table:

x	0	1	2	3
$f(x; 0)$.05	.05	.10	.80
$f(x; 1)$.05	.20	.45	.30

For testing $H_0 : \theta = 0$ against $H_1 : \theta = 1$, please list two different nonrandomized and two different randomized tests (any tests) of size .05. Compare these four tests (that is, is the first test more powerful than the second test, etc.?)

6. Consider a random sample of size n without replacement from a population of size N of which Np_1 are type A , Np_2 are type B , and $N(1 - p_1 - p_2)$ are type C . Let X_1 be the number of type A 's in the sample, and let X_2 be the number of type B 's.

(a) What is the joint density function of X_1 and X_2 ?

(b) Let $U = X_1 + X_2$. What are the mean and variance of U ?

(c) What is the conditional density function of $X_2 | U$?