

國立政治大學應用數學系九十三學年度第一學期研究生學科考試試題

科目：數理統計

1. Let  $X$  be a continuous random variable. Show that

$$EX = \int_0^{\infty} P(X > y) dy - \int_{-\infty}^0 P(X < y) dy.$$

2. Let  $X$  and  $Y$  be random variables. Show that

$$\text{Var}(Y) = E[\text{Var}(Y|X)] + \text{Var}[E(Y|X)].$$

3. Let  $(X, Y)$  have joint density function

$$f(x, y) = cxy, \quad 0 < y < x < 1.$$

Find  $\text{Var}(Y|X = 1/2)$ .

4. Let  $X_1, \dots, X_n$  be independent, with  $X_i \sim N(i\theta, 1)$ .

(a) Find the MLE of  $\theta$ .

(b) Find the best estimator of  $\theta$ .

5. Let  $X_1, \dots, X_n$  be independent with  $X_i \sim E(i\theta)$ ,  $\theta > 0$ . Find the lower bound for an unbiased estimator of  $\theta^{-1}$ .

6. (a) State Neyman-Pearson Theorem.

(b) Let  $X_1, \dots, X_n$  be independent, with  $X_i \sim P(\theta)$ . Find a UMP size- $\alpha$  test that  $\theta = 1$  against  $\theta > 1$ .