國立政治大學應用數學系九十四學年度第二學期研究生學科考試試題

科目：數理統計

1. (a) Consider a Poisson process with rate λ. Let X be the time till the second arrival. Find the density of X.

   (b) Suppose that we sample with replacement from a population of size N with proportion of successes p. Let V be the number of failures before the r-th success. Find the moment-generating function of V.

2. Let Z and U be independent, $Z \sim N(0,1)$, and $U \sim \chi_k^2$. Let $T = Z/\sqrt{U/k}$ and $W = U$.

   (a) Find the joint density of T and W.

   (b) Find the marginal density of T.

3. Let $X_1, \ldots, X_n$ be independent, with $X_i \sim N(\mu, \sigma^2)$. Find the size-$\alpha$ LRT for testing that $\sigma^2 = c^2$ against $\sigma^2 > c^2$, and show that it rejects the null hypothesis if $U > \chi_{n-1}^2(\alpha)$, where $U = (n-1)S^2/c^2$.

4. Let $X_1, \ldots, X_n$ be independent, with $X_i \sim E(i\theta)$.

   (a) Find the MLE of θ.

   (b) Find a $(1-\alpha)$ confidence interval for θ.

5. (Problem 4 continued)

   (a) Find the lower bound for an unbiased estimator of $1/\theta$.

   (b) Find the best unbiased estimator of θ^2.

6. (a) Let X be a discrete random variable having the following density function:

x	0	1	2	3	4	5
$f(x;0)$	.05	.05	.10	.10	.20	.50
$f(x;1)$	.10	.15	.25	.15	.25	.10

   Find the most powerful .15 test that $\theta = 1$ against $\theta = 0$.

   (b) State and prove Neymann-Pearson Theorem for the continuous case.