

國立政治大學應用數學系九十五學年度第二學期研究生學科考試試題

科目：數理統計

1. Let (X, Y) have joint density $f(x, y) = kx^2y^3, 0 < y < x < 1$.
 - (a) Find the marginal density of Y .
 - (b) Find $Var(X | Y = 2/3)$.
 2. (a) Consider a Poisson process with rate λ . Let T be the time till the third arrival. Find the density of T .
 - (b) Find the moment generating function of T .
 3. Let X_1, X_2, X_3 be independent and $X_i \sim P(m_i)$. Let $Y = X_1 + X_2 + X_3$, and $g = m_1 + m_2 + m_3$. Show that $(X_1, X_2) | Y \sim T(Y, (m_1/g, m_2/g))$.
 4. Let $(X, Y) \sim T(n, (\theta^2, 2\theta(1 - \theta)))$.
 - (a) Find the lower bound for an unbiased estimator of θ .
 - (b) Find the best unbiased estimator of θ .
 5. Let X_1, X_2, \dots, X_n be independent, with $X_i \sim E(i\theta)$. Show that the LRT for testing that $\theta = c$ against $\theta < c$ rejects the null hypothesis if $U = 2nR/c$ is too small, where $R = \sum_{i=1}^n X_i/n_i$.
 6. Let X_1, X_2, \dots, X_n be independent, with $X_i \sim E(i\theta)$.
 - (a) Find a $(1 - \alpha)$ confidence interval for θ , where $0 < \alpha < 1$.
 - (b) Find a UMP size- α test for testing $\theta = c$ against $\theta > c$.
- $(X_1, X_2) \sim T(n, (\theta_1, \theta_2)) \Leftrightarrow f(x_1, x_2) = \frac{n!}{x_1!x_2!(n-x_1-x_2)!} \theta_1^{x_1} \theta_2^{x_2} (1 - \theta_1 - \theta_2)^{n-x_1-x_2}, x_1, x_2 \geq 0, x_1 + x_2 \leq n$.
- $X \sim P(\lambda) \Leftrightarrow f(x) = e^{-\lambda} \lambda^x / x!, x = 0, 1, 2, \dots$
- $X \sim E(\theta) \Leftrightarrow f(x) = (1/\theta) e^{-x/\theta}, x > 0$.