國立政治大學應用數學系九十六學年度第一學期研究生學科考試試題

科目：數理統計

1. Let (X, Y) be jointly uniformly distributed on the triangle $0 < x < y < 1$. Let $U = X/Y$.
   (a) Find the marginal densities of X and Y.
   (b) Find the density of U.
2. Let (X, Y, Z) have joint moment-generating function $M(r, s, t) = (1 - r)^{-2}(1 + r - 2s)^{-3}(1 + 2r - s + 3t)^{-1}$.
   (a) Find $Cov(X, Y)$.
   (b) Find the joint moment-generating function of $U = 2X - Y$ and $V = Z - Y$. Are U and V independent?
3. (a) Let X, Y be random variables. Show that $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$.
   (b) Let (X, Y) have joint density $f(x, y) = cxy^2$, $0 < x < y < 1$. Find c and $Var(X|Y = 2/3)$.
4. Let $X_1, X_2, \ldots, X_n$ be independent with $X_i \sim E(i\theta)$, and let $R = \sum_{i=1}^{n} X_i/n_i$.
   (a) Find an unbiased estimator of θ^{-1}.
   (b) Is this estimator efficient?
5. (a) State the Neyman-Pearson theorem and prove it for the continuous case.
   (b) Let $X_1, \ldots, X_n$ be independent with $X_i \sim B(1, \theta)$. Find a UMP size-0.1 test that $\theta = 0.4$ against $\theta < 0.4$ when $n = 12$.
6. (a) Let $(X, Y) \sim T(n, (\theta^2, 2\theta(1 - \theta)))$. Find the lower bound for an unbiased estimator of θ^2.
   (b) Let X be a positive continuous random variable. Show that $E(X) = \int_0^\infty P(X > y)\, dy$.

TABLE A.2 (*continued*)

Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	.10	.20	.25	.30	.40	.50	.60	.70	.80	.90
12	0	.2824	.0687	.0317	.0138	.0022	.0002	.0000			
	1	.6590	.2749	.1584	.0850	.0196	.0032	.0003	.0000		
	2	.8891	.5583	.3907	.2528	.0834	.0193	.0028	.0002	.0000	
	3	.9744	.7946	.6488	.4925	.2253	.0730	.0153	.0017	.0001	
	4	.9957	.9274	.8424	.7237	.4382	.1938	.0573	.0095	.0006	.0000
	5	.9995	.9806	.9456	.8821	.6652	.3872	.1582	.0386	.0039	.0001
	6	.9999	.9961	.9857	.9614	.8418	.6128	.3348	.1178	.0194	.0005
	7	1.0000	.9994	.9972	.9905	.9427	.8062	.5618	.2763	.0726	.0043
	8		.9999	.9996	.9983	.9847	.9270	.7747	.5075	.2054	.0256
	9		1.0000	1.0000	.9998	.9972	.9807	.9166	.7472	.4417	.1109
	10				1.0000	.9997	.9968	.9804	.9150	.7251	.3410
	11					1.0000	.9998	.9978	.9862	.9313	.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000
13	0	.2542	.0550	.0238	.0097	.0013	.0001	.0000			
	1	.6213	.2336	.1267	.0637	.0126	.0017	.0001	.0000		
	2	.8661	.5017	.3326	.2025	.0579	.0112	.0013	.0001		
	3	.9658	.7473	.5843	.4206	.1686	.0461	.0078	.0007	.0000	
	4	.9935	.9009	.7940	.6543	.3530	.1334	.0321	.0040	.0002	
	5	.9991	.9700	.9198	.8346	.5744	.2905	.0977	.0182	.0012	.0000
	6	.9999	.9930	.9757	.9376	.7712	.5000	.2288	.0624	.0070	.0001
	7	1.0000	.9980	.9944	.9818	.9023	.7095	.4256	.1654	.0300	.0009
	8		.9998	.9990	.9960	.9679	.8666	.6470	.3457	.0991	.0065
	9		1.0000	.9999	.9993	.9922	.9539	.8314	.5794	.2527	.0342
	10			1.0000	.9999	.9987	.9888	.9421	.7975	.4983	.1339
	11				1.0000	.9999	.9983	.9874	.9363	.7664	.3787
	12					1.0000	.9999	.9987	.9903	.9450	.7458
	13						1.0000	1.0000	1.0000	1.0000	1.0000
14	0	.2288	.0440	.0178	.0068	.0008	.0001	.0000			
	1	.5846	.1979	.1010	.0475	.0081	.0009	.0001			
	2	.8416	.4481	.2811	.1608	.0398	.0065	.0006	.0000		
	3	.9559	.6982	.5213	.3552	.1243	.0287	.0039	.0002		
	4	.9908	.8702	.7415	.5842	.2793	.0898	.0175	.0017	.0000	
	5	.9985	.9561	.8883	.7805	.4859	.2120	.0583	.0083	.0004	
	6	.9998	.9884	.9617	.9067	.6925	.3953	.1501	.0315	.0024	.0000
	7	1.0000	.9976	.9897	.9685	.8499	.6047	.3075	.0933	.0116	.0002
	8		.9996	.9978	.9917	.9417	.7880	.5141	.2195	.0439	.0015
	9		1.0000	.9997	.9983	.9825	.9102	.7207	.4158	.1298	.0092
	10			1.0000	.9998	.9961	.9713	.8757	.6448	.3018	.0441
	11				1.0000	.9994	.9935	.9602	.8392	.5519	.1584
	12					.9999	.9991	.9919	.9525	.8021	.4154
	13					1.0000	.9999	.9992	.9932	.9560	.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000