

考試科目 Course	數理統計	開課系級 Dept. & Class	研究所	日期 Date, Period	99年9月20日 上午9:00~12:00	試題編號 Course No.	
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本試卷共有 6 個題目，

碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。

博士班：6 題全作答，每題 17 分，超過 100 分則以 100 分計。

1. (1) Let (X, Y) have joint density $f(x, y) = cxy^2, 0 < x < y < 1$
Find $\text{Var}(X | Y = 2/3)$.

(2) Suppose that $X \sim P(\lambda)$ and $Y|X \sim B(X, p)$.
Find the marginal density of Y .

2. Let X_1, \dots, X_n be independently identically distributed with X_i having density $f(x; \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$.

(1) Find the asymptotic distribution for the MLE $\hat{\theta}_n$ of θ .

(2) Find the best unbiased estimator of $1/\theta$.

3. (1) Let X, Y be random variables. Show that
 $\text{Var} Y = E \text{Var}(Y|X) + \text{Var} E(Y|X)$

(2) Let X_1, X_2, \dots be a sequence of random variables with $E X_n = \mu_n, \text{Var} X_n = \sigma_n^2$. Show that
 $\mu_n \rightarrow a, \sigma_n^2 \rightarrow 0 \Rightarrow X_n \xrightarrow{P} a$.

4. (1) Let X_1, \dots, X_n be independent, with $X_i \sim B(1, \theta)$

Find a UMP size-0.05 test that $\theta = 0.4$ against $\theta > 0.4$

本考試： 不需使用簡易計算機， 使用簡易計算機

← 請出題老師勾選，謝謝！

命題老師：
(Teacher)

年 9 月 17 日

試題隨卷繳交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。

Remarks : For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.

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when $n=10$.

(2) State Neyman - Pearson theorem.

5. (1) Let X_1, \dots, X_n be independent, with $X_i \sim N(0, \theta^2)$. Find the UMP size- α test for testing the null hypothesis that $\theta = 1$ against $\theta > 1$. If $\alpha = 0.05$, $n=4$, $X_1=2$, $X_2=1$, $X_3=-3$ and $X_4=-2$, do you accept or reject the null hypothesis with this test?

(2) Find a 95% confidence interval for θ by using the data given in (1).

6 (1) Let X_1, X_2, X_3 be iid with common pdf $f(x) = e^{-x}$, $x > 0$. Find the joint pdf of $Y_1 = X_1/X_2$, $Y_2 = X_3/(X_1 + X_2)$, and $Y_3 = X_1 + X_2$.

(2) Let (X, Y) be jointly uniformly distributed on the triangle $0 < x < y < 1$. Let $Z = X/Y$. Find the density of Z .

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命題老師：
(Teacher)

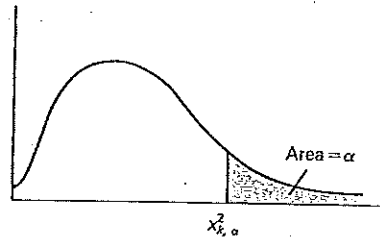
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TABLE 3 UPPER QUANTILES OF THE χ^2 DISTRIBUTION



k	$\chi_k^2 .995$	$\chi_k^2 .99$	$\chi_k^2 .975$	$\chi_k^2 .95$	$\chi_k^2 .90$	$\chi_k^2 .10$	$\chi_k^2 .05$	$\chi_k^2 .025$	$\chi_k^2 .01$	$\chi_k^2 .005$
1	.00	.00	.00	.00	.02	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.54	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2
500	422.3	429.4	439.9	449.1	459.9	540.9	553.1	563.9	576.5	585.2
1000	888.6	898.8	914.3	927.6	943.1	1058	1075	1090	1107	1119

Adapted from R.N. Goldman and J.S. Weinberg, *Statistics—An Introduction*. (Englewood Cliffs, N.J.: Prentice Hall, 1985), 681.

TABLE 5 BINOMIAL DISTRIBUTION FUNCTIONS. $P(X \leq x), X \sim B(n, p)$.

n	x	p												
		0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95
5	0	.774	.590	.328	.237	.168	.078	.031	.010	.002	.001	.000	.000	.000
	1	.977	.919	.737	.633	.528	.337	.188	.087	.031	.016	.007	.000	.000
	2	.999	.991	.942	.896	.837	.683	.500	.317	.163	.104	.058	.009	.001
	3	.999	.999	.993	.984	.969	.913	.813	.663	.472	.367	.263	.081	.023
	4	.999	.999	.999	.999	.998	.990	.969	.922	.832	.763	.672	.410	.226
	5	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
10	0	.599	.349	.107	.056	.028	.006	.001	.000	.000	.000	.000	.000	.000
	1	.914	.736	.376	.244	.149	.046	.011	.002	.000	.000	.000	.000	.000
	2	.988	.930	.678	.526	.383	.167	.055	.012	.002	.000	.000	.000	.000
	3	.999	.987	.879	.776	.650	.382	.172	.055	.011	.004	.001	.000	.000
	4	.999	.998	.967	.922	.850	.633	.377	.166	.047	.020	.006	.000	.000
	5	.999	.999	.994	.980	.953	.834	.623	.367	.150	.078	.033	.002	.000
	6	.999	.999	.999	.996	.989	.945	.828	.618	.350	.224	.121	.013	.001
	7	.999	.999	.999	.999	.998	.988	.945	.833	.617	.474	.322	.070	.012
	8	.999	.999	.999	.999	.999	.998	.989	.954	.851	.756	.624	.264	.086
	9	.999	.999	.999	.999	.999	.999	.999	.994	.972	.944	.893	.651	.401
	10	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
15	0	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000
	1	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000
	2	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000
	3	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000
	4	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000
	5	.999	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000
	6	.999	.999	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000
	7	.999	.999	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000
	8	.999	.999	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000
	9	.999	.999	.999	.999	.999	.991	.941	.783	.485	.314	.164	.013	.001
	10	.999	.999	.999	.999	.999	.999	.991	.941	.783	.485	.314	.164	.013
	11	.999	.999	.999	.999	.999	.999	.998	.982	.909	.703	.539	.352	.056
	12	.999	.999	.999	.999	.999	.999	.996	.973	.873	.764	.602	.184	.036
	13	.999	.999	.999	.999	.999	.999	.999	.995	.965	.920	.833	.451	.171
	14	.999	.999	.999	.999	.999	.999	.999	.999	.995	.987	.965	.794	.537
	15	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
20	0	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	1	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000
	2	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000
	3	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000
	4	.997	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000
	5	.999	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000
	6	.999	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000
	7	.999	.999	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000
	8	.999	.999	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000
	9	.999	.999	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000
	10	.999	.999	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000
	11	.999	.999	.999	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000
	12	.999	.999	.999	.999	.999	.979	.868	.584	.228	.102	.032	.000	.000