國立政治大學 應用數學系 九十九 學年度第 二 學期　學科 考試試題

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考試科目 Course	數理統計	開課系級 Dept, & Class	研究所	日期 Date, Period	100年3月7日 上午 9:00~12:00	試題編號 Course No.	

本試卷共有 6 個題目，
碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。
博士班：6 題全做答，每題 17 分，超過 100 分則以 100 分計。

1. (1) Suppose $X \sim P(\lambda)$ and the conditional distribution of Y given X is $Y|X \sim B(X, p)$. Find the marginal density of Y.

(2) Let X and Y be independently uniformly distributed on $(0,1)$. Let $U = X/Y$. Find the density function of U.

2. Let (X, Y) have joint density function

$$f(x,y) = \frac{n!}{x!\,y!\,(n-x-y)!}(\theta^2)^x(2\theta(1-\theta))^y((1-\theta)^2)^{n-x-y},$$

$x = 0, 1, 2, \cdots$, $y = 0, 1, 2, \cdots$, $x + y \le n$.

(1) Find the MLE $\hat{\theta}$ of θ.

(2) Find $Var(\hat{\theta})$.

3. Let $X_1, X_2, \cdots$ be a sequence of independent random variables with X_i uniformly distributed on the interval $(0,1)$. Let $U_n = \max_{i \le n} X_i$ and $W_n = n(1-U_n)$.

(1) Find $Var(U_n)$.

(2) Find the limiting distribution of W_n.

注意：在各題計算過程中，若有引用重要定理之結果，請敘述其「名稱」或「內容」。

本考試：■ 不需使用簡易計算機，□ 使用簡易計算機　←請出題老師勾選，謝謝！

命題老師: (Teacher)	（簽章）100 年 3 月 3 日 (Signature & date)	試題隨卷繳交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。

Remarks ：For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.

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4. Let $X_1, \cdots, X_n$ be independently identically distributed with X_i having density $f(x;\theta) = \theta(1-\theta)^x$, $x = 0, 1, 2, \cdots$, $0 < \theta < 1$.

(1) Find the lower bound for the variance of an unbiased estimator of θ^2.

(2) Find the best unbiased estimator of $\frac{1}{\theta}$.

5. (1) State the Neyman-Pearson theorem.

(2) Let X be a discrete random variable with density $f(x;\theta)$ given in the following table:

x	0	1	2	3	4	5
$f(x;0)$	0.05	0.05	0.1	0.1	0.2	0.5
$f(x;1)$	0.1	0.15	0.25	0.15	0.25	0.1

Find the most powerful size-0.1 test for testing that $\theta = 0$ against $\theta = 1$.

6. Let $X_1, \cdots, X_n$ be independent, with $X_i \sim N(1, \theta^2)$.

(1) Find a $100(1-\alpha)\%$ confidence interval for θ.

(2) Find the UMP size-α test for testing $H_0: \theta = 1$ against $H_1: \theta > 1$. If $\alpha = 0.05$, $n = 4$, $X_1 = 3$, $X_2 = 2$, $X_3 = 1$ and $X_4 = -2$, do you accept or reject the null hypothesis with this test?

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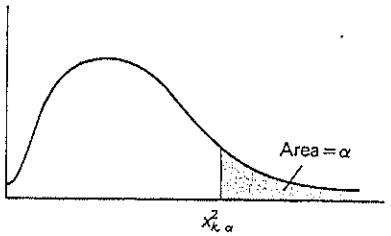
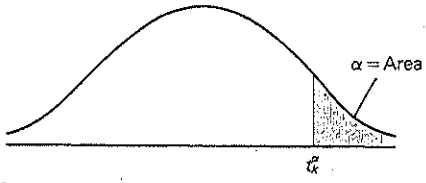
TABLE 3 UPPER QUANTILES OF THE χ^2 DISTRIBUTION



k	$\chi^2_{k,.995}$	$\chi^2_{k,.99}$	$\chi^2_{k,.975}$	$\chi^2_{k,.95}$	$\chi^2_{k,.90}$	$\chi^2_{k,.10}$	$\chi^2_{k,.05}$	$\chi^2_{k,.025}$	$\chi^2_{k,.01}$	$\chi^2_{k,.005}$
1	.00	.00	.00	.00	.02	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.54	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2
500	422.3	429.4	439.9	449.1	459.9	540.9	553.1	563.9	576.5	585.2
1000	888.6	898.8	914.3	927.6	943.1	1058	1075	1090	1107	1119

TABLE 2 UPPER QUANTILES OF THE t-DISTRIBUTION



k	$t_k^{.25}$	$t_k^{.2}$	$t_k^{.15}$	$t_k^{.1}$	$t_k^{.05}$	$t_k^{.025}$	$t_k^{.01}$	$t_k^{.005}$
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	.817	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	.765	.978	1.250	1.638	2.353	3.183	4.541	5.841
4	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604
5	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032
6	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707
7	.711	.896	1.119	1.415	1.895	2.365	2.998	3.500
8	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355
9	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250
10	.700	.879	1.093	1.372	1.813	2.228	2.764	3.169
11	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106
12	.696	.873	1.083	1.356	1.782	2.179	2.681	3.055
13	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012
14	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977
15	.691	.866	1.074	1.341	1.753	2.132	2.602	2.947
16	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921
17	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898
18	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878
19	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861
20	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845
21	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831
22	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819
23	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807
24	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797
25	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787
26	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779
27	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771
28	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763
29	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756
30	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750
31	.683	.854	1.054	1.310	1.696	2.040	2.453	2.744
32	.682	.853	1.054	1.309	1.694	2.037	2.449	2.739
33	.682	.853	1.053	1.308	1.692	2.035	2.445	2.733
34	.682	.852	1.053	1.307	1.691	2.032	2.441	2.728
35	.682	.852	1.052	1.306	1.690	2.030	2.438	2.724
36	.681	.852	1.052	1.306	1.688	2.028	2.434	2.720
37	.681	.852	1.051	1.305	1.687	2.026	2.431	2.716
38	.681	.851	1.051	1.304	1.686	2.024	2.428	2.712
39	.681	.851	1.050	1.304	1.685	2.023	2.426	2.708
40	.681	.851	1.050	1.303	1.684	2.021	2.423	2.705
50	.679	.849	1.047	1.299	1.676	2.009	2.403	2.678
60	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660
70	.678	.847	1.044	1.294	1.667	1.995	2.381	2.648
80	.678	.846	1.043	1.292	1.664	1.990	2.374	2.639
90	.677	.846	1.043	1.291	1.662	1.987	2.368	2.632
100	.677	.845	1.042	1.290	1.660	1.984	2.364	2.626