

考試科目 Course	數理統計	開課系級 Dept. & Class	研究所	日期 Date, Period	100年3月7日 上午9:00~12:00	試題編號 Course No.
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本試卷共有 6 個題目，
 碩士班：請選 5 題作答，每題 20 分，請在答案卷最前面註明所選的 5 題，否則依學生作答之前 5 題計分。
 博士班：6 題全作答，每題 17 分，超過 100 分則以 100 分計。

1. (1) Suppose $X \sim P(\lambda)$ and the conditional distribution of Y given X is $Y|X \sim B(X, p)$. Find the marginal density of Y .
- (2) Let X and Y be independently uniformly distributed on $(0, 1)$. Let $U = X/Y$. Find the density function of U .

2. Let (X, Y) have joint density function

$$f(x, y) = \frac{n!}{x!y!(n-x-y)!} (\theta^2)^x (2\theta(1-\theta))^y (1-\theta)^{n-x-y},$$

$$x=0, 1, 2, \dots, y=0, 1, 2, \dots, x+y \leq n.$$

- (1) Find the MLE $\hat{\theta}$ of θ .
- (2) Find $\text{Var}(\hat{\theta})$.

3. Let X_1, X_2, \dots be a sequence of independent random variables with X_i uniformly distributed on the interval $(0, 1)$.

$$\text{Let } U_n = \max_{i \leq n} X_i \text{ and } W_n = n(1 - U_n).$$

- (1) Find $\text{Var}(U_n)$.
- (2) Find the limiting distribution of W_n .

注意：在各題計算過程中，若有引用重要定理之結果，請敘述其名稱或內容。

本考試： 不需使用簡易計算機， 使用簡易計算機

← 請出題老師勾選，謝謝！

命題老師：
(Teacher)

(簽章) 100 年 3 月 3 日
(Signature & date)

試題隨卷繳交

命題紙使用說明：試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。

Remarks: For the convenience of reprinting please Write questions in black or blue-black (but no red) ink.

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4. Let X_1, \dots, X_n be independently identically distributed with X_i having density $f(x; \theta) = \theta(1-\theta)^x$, $x=0, 1, 2, \dots$, $0 < \theta < 1$.

(1) Find the lower bound for the variance of an unbiased estimator of θ^2 .

(2) Find the best unbiased estimator of $\frac{1}{\theta}$.

5. (1) State the Neyman-Pearson theorem.

(2) Let X be a discrete random variable with density $f(x; \theta)$ given in the following table:

x	0	1	2	3	4	5
$f(x; 0)$	0.05	0.05	0.1	0.1	0.2	0.5
$f(x; 1)$	0.1	0.15	0.25	0.15	0.25	0.1

Find the most powerful size - 0.1 test for testing that $\theta=0$ against $\theta=1$.

6. Let X_1, \dots, X_n be independent, with $X_i \sim N(1, \theta^2)$.

(1) Find a $100(1-\alpha)\%$ confidence interval for θ .

(2) Find the UMP size - α test for testing $H_0: \theta = 1$ against

$H_1: \theta > 1$. If $\alpha = 0.05$, $n = 4$, $X_1 = 3$, $X_2 = 2$, $X_3 = 1$ and $X_4 = -2$,

do you accept or reject the null hypothesis with this test?

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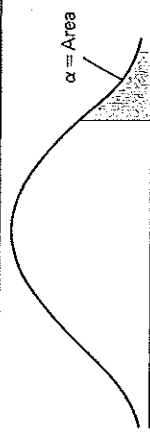
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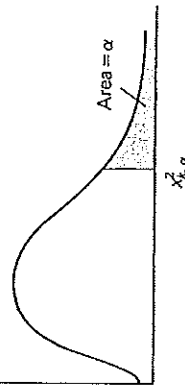
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TABLE 2 UPPER QUANTILES OF THE t-DISTRIBUTION



k	$t_k^{.95}$	$t_k^{.90}$	$t_k^{.85}$	$t_k^{.80}$	$t_k^{.75}$	$t_k^{.70}$	$t_k^{.65}$	$t_k^{.60}$	$t_k^{.55}$	$t_k^{.50}$	$t_k^{.45}$	$t_k^{.40}$	$t_k^{.35}$	$t_k^{.30}$	$t_k^{.25}$	$t_k^{.20}$	$t_k^{.15}$	$t_k^{.10}$	$t_k^{.05}$
1	1.000	1.376	1.633	1.863	2.078	2.278	2.457	2.617	2.757	2.878	2.977	3.057	3.119	3.172	3.219	3.263	3.305	3.346	3.385
2	.817	1.061	1.286	1.478	1.628	1.747	1.846	1.926	1.989	2.047	2.100	2.149	2.195	2.239	2.281	2.321	2.359	2.396	2.432
3	.765	.978	1.190	1.341	1.460	1.551	1.626	1.687	1.736	1.783	1.828	1.871	1.912	1.951	1.989	2.026	2.062	2.097	2.131
4	.741	.941	1.136	1.267	1.371	1.451	1.514	1.566	1.612	1.655	1.696	1.735	1.772	1.808	1.843	1.878	1.911	1.944	1.976
5	.727	.920	1.110	1.229	1.321	1.391	1.445	1.491	1.532	1.571	1.609	1.645	1.679	1.713	1.746	1.778	1.810	1.841	1.871
6	.718	.906	1.093	1.199	1.281	1.343	1.390	1.430	1.466	1.500	1.533	1.565	1.597	1.629	1.660	1.690	1.719	1.748	1.776
7	.711	.896	1.080	1.182	1.259	1.314	1.355	1.393	1.430	1.461	1.491	1.520	1.548	1.575	1.601	1.627	1.652	1.677	1.701
8	.706	.889	1.070	1.168	1.241	1.291	1.331	1.367	1.400	1.431	1.459	1.486	1.512	1.537	1.561	1.584	1.607	1.629	1.651
9	.703	.883	1.062	1.157	1.226	1.274	1.312	1.346	1.377	1.406	1.433	1.458	1.482	1.505	1.527	1.548	1.568	1.587	1.606
10	.700	.879	1.057	1.149	1.214	1.259	1.295	1.327	1.356	1.383	1.408	1.431	1.453	1.474	1.494	1.513	1.531	1.549	1.566
11	.697	.876	1.052	1.141	1.203	1.245	1.279	1.309	1.336	1.361	1.384	1.406	1.426	1.445	1.462	1.479	1.495	1.511	1.527
12	.696	.873	1.048	1.135	1.194	1.234	1.267	1.295	1.321	1.345	1.367	1.388	1.407	1.424	1.440	1.455	1.470	1.484	1.499
13	.694	.870	1.045	1.130	1.187	1.226	1.257	1.284	1.308	1.330	1.351	1.371	1.389	1.405	1.420	1.434	1.448	1.461	1.475
14	.692	.868	1.042	1.125	1.181	1.219	1.248	1.273	1.296	1.317	1.336	1.354	1.371	1.387	1.402	1.415	1.428	1.440	1.452
15	.691	.866	1.040	1.122	1.177	1.214	1.242	1.266	1.288	1.308	1.326	1.343	1.359	1.374	1.388	1.401	1.414	1.425	1.437
16	.690	.865	1.038	1.119	1.173	1.209	1.236	1.259	1.280	1.299	1.316	1.332	1.347	1.361	1.374	1.386	1.398	1.409	1.420
17	.689	.863	1.036	1.116	1.169	1.204	1.231	1.254	1.274	1.292	1.308	1.323	1.337	1.350	1.362	1.374	1.385	1.396	1.406
18	.688	.862	1.034	1.113	1.165	1.200	1.226	1.249	1.268	1.285	1.300	1.314	1.327	1.340	1.352	1.363	1.374	1.384	1.394
19	.688	.861	1.032	1.111	1.163	1.198	1.224	1.247	1.266	1.282	1.297	1.310	1.322	1.334	1.345	1.356	1.366	1.376	1.386
20	.687	.860	1.030	1.109	1.161	1.196	1.221	1.244	1.263	1.278	1.292	1.305	1.317	1.328	1.339	1.349	1.359	1.368	1.377
21	.686	.859	1.028	1.107	1.158	1.193	1.218	1.240	1.259	1.274	1.288	1.300	1.311	1.322	1.332	1.342	1.351	1.360	1.369
22	.686	.858	1.026	1.105	1.156	1.191	1.216	1.238	1.256	1.271	1.284	1.296	1.307	1.317	1.327	1.336	1.345	1.354	1.363
23	.685	.858	1.024	1.103	1.154	1.189	1.214	1.236	1.254	1.269	1.282	1.293	1.303	1.313	1.322	1.331	1.340	1.348	1.356
24	.685	.857	1.022	1.101	1.152	1.187	1.212	1.234	1.252	1.267	1.279	1.290	1.300	1.310	1.319	1.328	1.336	1.344	1.352
25	.684	.856	1.020	1.099	1.150	1.185	1.210	1.232	1.250	1.264	1.276	1.286	1.296	1.305	1.314	1.323	1.331	1.339	1.347
26	.684	.856	1.018	1.097	1.148	1.183	1.208	1.230	1.248	1.262	1.273	1.283	1.292	1.301	1.310	1.318	1.326	1.334	1.342
27	.684	.855	1.016	1.095	1.146	1.181	1.206	1.228	1.246	1.259	1.270	1.280	1.289	1.297	1.306	1.314	1.322	1.330	1.337
28	.683	.855	1.014	1.093	1.144	1.179	1.204	1.226	1.244	1.257	1.267	1.276	1.285	1.293	1.301	1.309	1.316	1.324	1.331
29	.683	.854	1.012	1.091	1.142	1.177	1.202	1.224	1.242	1.255	1.265	1.274	1.282	1.290	1.298	1.306	1.313	1.320	1.327
30	.683	.854	1.010	1.089	1.140	1.175	1.200	1.222	1.240	1.253	1.263	1.272	1.280	1.288	1.295	1.303	1.310	1.317	1.324
31	.683	.854	1.008	1.087	1.138	1.173	1.198	1.220	1.238	1.251	1.260	1.268	1.276	1.283	1.290	1.297	1.304	1.311	1.318
32	.682	.853	1.006	1.085	1.136	1.171	1.196	1.218	1.236	1.249	1.258	1.266	1.273	1.280	1.287	1.293	1.299	1.305	1.311
33	.682	.853	1.004	1.083	1.134	1.169	1.194	1.216	1.234	1.247	1.256	1.264	1.271	1.277	1.283	1.289	1.295	1.301	1.307
34	.682	.852	1.002	1.081	1.132	1.167	1.192	1.214	1.232	1.245	1.254	1.262	1.269	1.275	1.281	1.286	1.291	1.296	1.302
35	.682	.852	1.000	1.079	1.130	1.165	1.190	1.212	1.230	1.243	1.252	1.259	1.266	1.272	1.277	1.282	1.287	1.292	1.297
36	.681	.852	1.000	1.077	1.128	1.163	1.188	1.210	1.228	1.241	1.249	1.256	1.262	1.268	1.273	1.278	1.283	1.288	1.292
37	.681	.852	1.000	1.076	1.127	1.162	1.187	1.209	1.227	1.239	1.247	1.254	1.260	1.265	1.270	1.275	1.279	1.284	1.288
38	.681	.851	1.000	1.075	1.126	1.161	1.186	1.208	1.226	1.238	1.246	1.253	1.259	1.264	1.269	1.273	1.277	1.281	1.285
39	.681	.851	1.000	1.074	1.125	1.160	1.185	1.207	1.225	1.237	1.245	1.252	1.258	1.263	1.268	1.272	1.276	1.280	1.284
40	.681	.851	1.000	1.073	1.124	1.159	1.184	1.206	1.224	1.236	1.244	1.251	1.257	1.262	1.266	1.270	1.274	1.278	1.282
50	.679	.849	1.000	1.072	1.123	1.158	1.183	1.205	1.223	1.235	1.243	1.250	1.255	1.260	1.264	1.268	1.271	1.275	1.278
60	.679	.848	1.000	1.071	1.122	1.157	1.182	1.204	1.222	1.234	1.242	1.249	1.254	1.258	1.262	1.266	1.269	1.272	1.275
70	.678	.847	1.000	1.070	1.121	1.156	1.181	1.203	1.221	1.233	1.241	1.247	1.252	1.256	1.260	1.263	1.266	1.269	1.272
80	.678	.846	1.000	1.069	1.120	1.155	1.180	1.202	1.220	1.232	1.240	1.246	1.251	1.255	1.259	1.262	1.265	1.268	1.271
90	.677	.846	1.000	1.068	1.119	1.154	1.179	1.201	1.219	1.231	1.238	1.244	1.249	1.253	1.257	1.260	1.263	1.266	1.269
100	.677	.845	1.000	1.067	1.118	1.153	1.178	1.200	1.218	1.230	1.237	1.242	1.247	1.251	1.254	1.257	1.260	1.263	1.266

TABLE 3 UPPER QUANTILES OF THE χ^2 DISTRIBUTION

k	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.85}$	$\chi^2_{.80}$	$\chi^2_{.75}$	$\chi^2_{.70}$	$\chi^2_{.65}$	$\chi^2_{.60}$	$\chi^2_{.55}$	$\chi^2_{.50}$	$\chi^2_{.55}$	$\chi^2_{.50}$	$\chi^2_{.45}$	$\chi^2_{.40}$	$\chi^2_{.35}$	$\chi^2_{.30}$	$\chi^2_{.25}$
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.010	.020	.050	.100	.210	.461	.708	1.055	1.385	1.676	1.920	2.179	2.446	2.700	2.953	3.203	3.450	3.693	3.933	4.171
3	.076	.115	.158	.200	.270	.337	.401	.463	.523	.581	.637	.691	.743	.794	.844	.893	.940	.986	1.031	1.075
4	.216	.300	.377	.450	.520	.587	.652	.715	.776	.835	.892	.948	1.003	1.056	1.108	1.159	1.209	1.257	1.304	1.350
5	.412	.557	.676	.770	.850	.919	.986	1.051	1.114	1.175	1.234	1.291	1.347	1.402	1.455	1.507	1.558	1.608	1.656	1.704
6	.583	.759	.895	1.000	1.080	1.149	1.215	1.279	1.341	1.401	1.459	1.516	1.571	1.625	1.678	1.730	1.781	1.830	1.878	1.925
7	.768	.965	1.119	1.230	1.300	1.369	1.434	1.497	1.558	1.617	1.674	1.730	1.784	1.837	1.889	1.940	1.990	2.039	2.087	2.134
8	.956	1.176	1.339	1.460	1.530	1.599	1.663	1.725	1.785	1.843	1.899	1.954	2.008	2.060	2.111	2.161	2.210	2.258	2.305	2.352
9	1.148	1.385	1.557	1.688	1.758	1.827	1.891	1.952	2.011	2.068	2.124	2.179	2.232	2.284	2.335	2.385	2.434	2.481	2.528	2.574
10	1.357	1.603	1.783	1.924	2.004	2.073	2.137	2.197	2.254	2.309	2.363	2.416	2.468	2.519	2.569	2.618	2.666	2.713	2.759	2.804
11	1.579	1.833	2.021	2.172	2.262	2.331	2.395	2.454	2.510	2.564	2.617	2.669	2.720	2.770	2.819	2.867	2.914	2.960	2.995	3.040
12	1.813	2.074	2.269	2.420	2.510	2.579	2.643													